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Effect of Transient Creep on the Collapse Time of Cylinders and Cones under External Pressure

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Nomenclature

A = cross-sectional area, in.^{2}
E = elastic tensile modulus, ksi
h = thickness of solid wall, or distance between facings in sandwich wall, in.
P = applied load, lb
s = material constant, hr^{-1}-deg^{-1}-°K
ΔH = material constant, cal-mol^{-1}
σ_0 = material constant, ksi
T = temperature, °K
M = temperature constant = $2sTe^{-\Delta H/RT}$ hr^{-1}
t = time, hr
α = nondimensional material constant for transient creep defined in text as $E_{\text{I}}/(E_{\text{I}} + E_{\text{II}})$
β = nondimensional material constant for transient creep defined in text as $M_{\text{I}}/M_{\text{II}}$
w = deflection, in.
ϵ = strain
σ = stress, ksi
$\bar{\sigma}$ = mean stress = P/A, ksi
σ_{cr} = elastic critical stress, ksi
τ = time constant, hr

Subscript

I,II = referring to different parts of the material
1,2 = referring to different sandwich facings

Introduction

MEASUREMENTS of the creep collapse times of long cylindrical shells under external pressure[1] and of finite cylindrical shells and cones[2,3] have been reported in the literature.

On the assumption that creep collapse occurs as a result of growth of the initial imperfections with time, a theory using steady-state creep has been developed, and a formula for the time to collapse has been derived. Comparisons of these theoretical results with experiment have been made[1–4] which showed the basic soundness of the theoretical foundation. In Ref. 4, for example, it was found that columns and cylindrical shells, with their widely different initial imperfections, could be handled by the same formula and plotted on the same chart.

The accuracy of the prediction of collapse time, however, in some cases left something to be desired. It was felt that one of the principal reasons for this lack of accuracy was the neglect of the transient component of creep. The theory assumed a constant creep rate throughout, whereas it was inevitable that the creep rates would be much higher than this constant value for a time following the application of external pressure. The question arose as to how much the time to collapse would be affected by the inclusion of the transient component of creep.

Fortunately, the mechanism for handling this problem was available.[5] Although three constants are sufficient to determine completely the steady-state creep, two additional constants, making five in all, are required to include the transient component. This generalization was employed in the attempt to improve the accuracy of collapse time predictions.

Analysis

In Ref. 5, a phenomenological theory was developed to account for the phenomena of transient creep. The distinguishing characteristic of this theory is that after a time the creep becomes steady, and so the theory includes within itself both the transient and the steady creep. The theory shows that by knowledge of five constants, it is possible to describe the entire family of creep curves for any material up to the beginning of tertiary creep. Three of the constants are those associated with the steady state of creep, whereas the remaining two serve to specify the transient state of creep. In Ref. 5, it was shown that the families of creep curves for four different materials (pure aluminum, gamma iron, lead, and 7075-T6 aluminum alloy) could be described completely by these five constants.

The theory assumes that any given material is split into two parts, each of which is itself viscoelastic with its own elasticity and its own viscosity. It is the interaction of these two parts which produces the transient creep. As time progresses, the interaction between the parts becomes of less and less importance, until the interaction substantially ceases, and the material then creeps at a steady rate.

The notation is the same as that used for steady state creep, with the addition of subscripts I and II for the two parts of

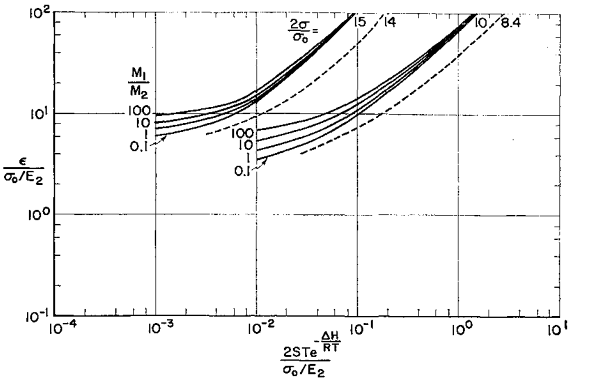

Fig. 1 Transient creep of 3003-H14 aluminum alloy shown as dashed curves superimposed on Fig. 1a of Ref. 5.

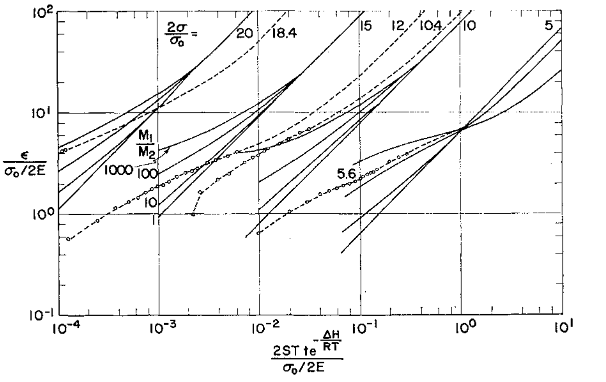

Fig. 2 Transient creep of 6061-T6 aluminum alloy shown as dashed curves superimposed on Fig. 1b of Ref. 5.

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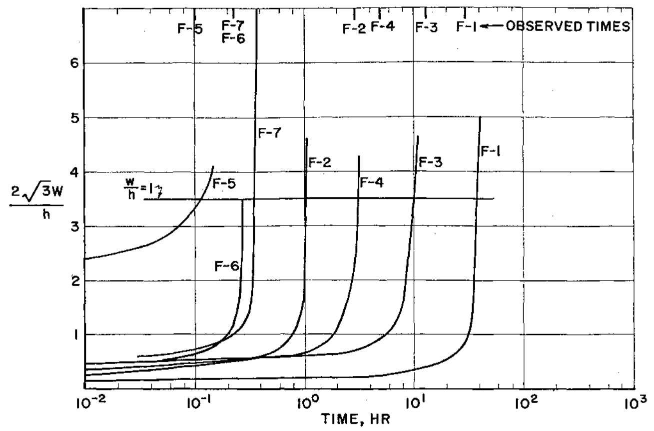

Fig. 3 Deflection curves for fixed end cylinders.

the material and of the new constants α and β defined as follows.

Equation (A-18) of Ref. 5 gives the general relation between strain rate $\dot{\epsilon}$, stress σ, time t, and temperature T as

$$\dot{\epsilon} = \frac{1}{E}\frac{d\sigma}{dt} + M \times \frac{\alpha[\beta + \cosh(2\sigma/\sigma_0)] - (1-\alpha)[(1/\beta) + \cosh(2\sigma/\sigma_0)]}{\cosh\sigma/\sigma_0} \cdot \frac{re^{-t/\tau}}{1 - r^2e^{-2t/\tau}} + \sinh\sigma/\sigma_0 \cdot \frac{1 + r^2e^{-2t/\tau}}{1 - r^2e^{-2t/\tau}} \quad (1)$$

where

$$\left.\begin{aligned} r &= \tanh\left[\frac{\alpha\sigma}{\sigma_0} - \frac{1}{2}\tanh^{-1}\frac{\sinh(2\sigma/\sigma_0)}{\beta + \cosh(2\sigma/\sigma_0)}\right] \\ \tau &= \frac{[\sigma_0/(M_{\rm I}M_{\rm II})^{1/2}][1/E_{\rm I}) + (1/E_{\rm II})]}{[\beta + (1/\beta) + 2\cosh(2\sigma/\sigma_0)]^{1/2}} \\ \alpha &= \frac{E_{\rm II}}{E_{\rm I} + E_{\rm II}} \qquad \beta = \frac{M_{\rm I}}{M_{\rm II}} \\ M &= \frac{2(M_{\rm I}M_{\rm II})^{1/2}\cosh(\sigma/\sigma_0)}{[\beta + (1/\beta) + 2\cosh(2\sigma/\sigma_0)]^{1/2}} \end{aligned}\right\} \quad (2)$$

The quantities α and β specify the transient creep. If $\alpha = \frac{1}{2}$, $\beta = 1$, then $r = 0$, the transient creep disappears, and the strain rate reduces to

$$\dot{\epsilon} = (1/E)(d\sigma/dt) + M\sinh(\sigma/\sigma_0) \quad (3)$$

which is the strain rate for the steady state [Eq. (I) of Ref. 4].

Analysis of the behavior of the structure under pressure and elevated temperature proceeds exactly as in Refs. 1–4 but uses Eq. (1) instead of Eq. (3). The analysis results in the expression for deflection rate for an infinitely long cylinder with solid walls:

$$\frac{d}{dt}\left(\frac{3^{1/2}w}{h/2}\right) = \frac{\frac{1}{2}(\sigma_0/\sigma_{cr})}{(\sigma_0/EM)[1 - (\bar{\sigma}/\sigma_{cr})]}\left\{A_1\frac{r_1e^{-t/\tau_1}}{1 - r_1^2e^{-2t/\tau_1}} - A_2\frac{r_2e^{-1/\tau_2}}{1 - r_2^2e^{-2t/\tau^2}} + \sinh\frac{\sigma_1}{\sigma_0}\cdot\frac{1 + r_1^2e^{-2t/\tau_1}}{1 + r_1^2e^{-2t/\tau_1}} - \sinh\frac{\sigma_2}{\sigma_0}\cdot\frac{1 + r_2^2e^{-2t/\tau_2}}{1 - r_2^2e^{-2t/\tau_2}}\right\} \quad (4)$$

where

$$A_1 = \frac{\alpha[\beta + \cosh(2\sigma_1/\sigma_0)] - (1-\alpha)\ [(1/\beta) + \cosh(2\sigma_1/\sigma_0)]}{\cosh(\sigma_1/\sigma_0)}$$

$$A_2 = \frac{\alpha[\beta + \cosh(2\sigma_2/\sigma_0)] - (1-\alpha)\ [(1/\beta) + \cosh(2\sigma_2/\sigma_0)]}{\cosh(\sigma_2/\sigma_0)}$$

Table 1 Summary of probable errors in collapse times

Type of structure	*PE*, collapse time: Steady-state solution, $w/h = \infty$, %	Transient creep solution, $w/h = 1$, %	$w/h = 3$, %
Cylinders, free ends	1150		358
Cylinders, simply supported ends	855	88	
Cylinders, fixed ends	68	28	
Cones, simply supported ends	115	48	

Equation (4) may be solved by a step-by-step process, beginning with the known initial imperfection w_0 at $t = 0$.

Materials

References 1–3 provide experimental data on both cylinders and cones under pressure. The material for the "infinitely long" tubes was 3003H-14 aluminum alloy with the following constants: $\Delta H = 35{,}000$ cal-mol^{-1}, $\sigma_0 = 714$ psi, $s = 2.3 \times 10^6$ hr^{-1}-deg-$^{-1}$ °K, $\alpha = 0.99$, $\beta = 1$.

The material for the cylinders of finite length and for the cones was 6061-T6 aluminum alloy with the following constants: $\Delta H = 39{,}000$ cal-mol^{-1}, $\sigma_0 = 2.5$ ksi, $s = 2.87 \times 10^7$ hr^{-1}-deg^{-1}-°K for the cylinders and 0.87×10^7 hr^{-1} deg^{-1}-°K for the cones, $\alpha = 0.5$, $\beta = 100$.

The first three constants are obtained from measurements of the minimum creep rate. The values of α and β are obtained from transient creep data by comparing these data with the curve families in Fig. 1 of Ref. 5. For this comparison, we have $E_1/E_2 = \alpha/(1-\alpha)$, $M_1/M_2 = \beta$. Figure 1 shows the comparison for the 3003 H-14 alloy, and Fig. 2 for the 6061-T6 alloy.

Results

The result of step-by-step integration of Eq. (4), starting with the known initial imperfection at $t = 0$, is a curve of deflection w against time. The character of the curves is shown in Fig. 3, which presents the computed curves for seven fixed-end cylinders. There occurs a time for each cylinder when the curve rises steeply; this time is taken to be the collapse time. A line has been drawn at $w/h = 1$ to make the time definite. The observed times are shown at the top of the figure. It is seen that the computed times appear in the same order as the observed times and at nearly the correct values.

Similar calculations have been made by means of the Institute GE 225 computer for 44 shell structures, seven of them cones, for which data have been obtained. In each case the calculated collapse time was compared with the observed time and the probable error computed. The improvement in prediction of collapse time using the transient creep solution is clearly shown in Table 1.

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